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CLASSICAL LOGIC: FORMALIZATION OF LOGICAL IMPLICATION

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ABSTRACT

The question of formulation on a pro forma basis (without analysing the specific content of variables) of the logical implication «A⊧B» is not still finally answered. This is due to the known difficulties. Firstly, the logical implication takes place in conditional (implicative) statements which, if they are true, may express logical implication or may not. Secondly, conditional statements can be true even when their antecedents and consequents are not related to each other according to their senses. Thirdly, the problem of what is called "para-doxes of material implication" is imposed on answering the question of the logical implication: it is known that the implication is true, as in the case if the falsity implicates the truth ("Truth follows from anything") as in the case when the arbitrary expression is derived from the contradiction ("contradiction (sequitur) quodlibet (ECQ)" ("A contradiction yields to completely contrary explanations")..

Keywords:

INTRODUCTION

DEFINITIONS OF LOGICAL SEQUENCE IN THE LITERATURE

Each logician offers his/her own solution to the problem of logical implication, or agrees with some already formulated definition. Some scientists believe that the problem of logical implication is insoluble within the framework of classical logic and are trying to solve this problem within the framework of non-classical logics, for example, in relevant logic.

In our opinion, the most adequate definition of logical implication was given by the domestic scientist E.K. Voishvillo, who said: "First of all, it seems that there is a generally accepted characteristic of this relationship between statements as a value independent of the meanings related to the descriptive terms of statements. In other words, we are talking about the relationship be-tween statements, depending only on their logical forms, or, more precisely, the relationship be-tween the logical contents of statements. Secondly, an important and generally recognized characteristic of the relationship A⊧B is that B is always true, when true A for any values of the descriptive terms in the logical forms A and B. The third main feature of the relationship of interest to us is already revealed from here: it takes place if and only if the logical content of B is part of the logical content of A" (Voishvillo, 2011). Fully agreeing with the ideas of E.K. Voishvillo, we note that here we are talking more about the properties of logical implication than about its definition. One way or another, Voishvillo does not formulate an effective algorithm for the "recognition" of logical implication in implicit statements, as, in fact, other scientists do not formulate it.

In another textbook, the same author writes: "The following relation between the formulas A⊧B takes place, that is, for any interpretation of the descriptive terms in A and B and for any attributions of values to free variables, the second is true if the first is true, in other words, first is false or second is true. This means that, firstly, if some descriptive term is somehow interpreted in A, then it is interpreted in B in the same way (of course, if it exists in this formula), and secondly, all free occurrences of the same variable in A and B are assigned to the same value. The statement B0 follows from the set of statements G0 if and only if this relation takes

place between the set of formulas G and B, respectively, which are the logical forms of the said statements. The last relation G⊧B is valid, i.e., the structure of G contains a finite subset of formulas A1, ... An ($n \geq 1$) such that (A1 &, ... & An)⊧B. The last relation, as in the state mental logic, is equivalent to the fact that the set of statements A1, ... An implies B, which in turn points to the property of the following relation noted earlier in the state mental logic, which consists in that if a certain statement follows from a certain set of statements, then it is also a consequence of any extension of this set " (Voishvillo & Degtyarev, 2013).

Other definitions are even more vague from the point of view of the final solution to the issue of logical implication, for example: "In formalized logical theories (calculi), the expression G⊧B means that the formula B of this calculus within the framework of the accepted semantics ... is true ... always when all formulas from G are true .... " (http://gtmarket.ru).

There are known attempts to present a logical implication as an implicit inference with single variables in the antecedent and consequent. However, uniform variables do not yet guarantee a logical implication ("If Paris is not the capital of Russia, then Moscow is the capital of Russia"), and in statements where there are no such common variables, a logical implication is possible ("If Euclid was wrong, then the parallel lines intersect").

Working hypothesis: The logical implication relationship "A⊧B" is the relationship between events "A" and "B", in which the occurrence of event "B" is more likely when "A" is present than without it ("A" confirms the event "B").

Materials and methods

The paper uses the building of truth tables and the Willard Van Orman Quine natural inference system of the Q3 type. Those works of philosophers and mathematicians are analysed, which touch upon the problem of formalization of logical implication: Voishvillo E. K. (1988), Zaitsev D. V., Sidorenko E. A. (Zaitsev & Sidorenko, 2001), Lobovikov V. O. (2015), Routley R., Muyer R. (Routley & Muyer, 1981), Smirnov V. A. (Smirnov, 1970), Tarski A. O (Tarsky, 2015), Shalak V. I. O (2007), Ackerman W. (1956), Lewis C.I., Langford S. H. (1932), Newton I. (1994), etc

Results

The imaginary "paradoxes of material implication"

The so-called "paradoxes" of material implication are based on an intuitive attempt to consider implication precisely as a logical implication of “A⊃B” ⇒ “A⊨B”. If we consider an implication based on its definition, it becomes clear that the implication “A⊃B” is a conditional expression that reflects the entire range of conditions “A” for the event “B”.

This range of conditions is very wide. These are also sufficient conditions when “B” arises both under condition “A” and without it; these are necessary conditions when “B” is impossible without “A”; these are both necessary and sufficient conditions when “A” and “B” are equivalent (identical).

If all this is borne in mind, then the “paradoxical” true statement “If the Moon is made of green cheese, then 2x2 = 4” becomes not so paradoxical. If we said that “If the Moon is made from green cheese, then 2x2 = 4 logically follows from this”, we would be wrong. But we only affirm that "2x2 = 4 is true, even if the Moon is made from green cheese." In other words, 2x2 = 4 is true under any conditions. That is why truth can be implicated (but not logically substantiated!) from anything.

Another “paradox” of implication is known as “anything follows from a contradiction”. That is, if we have “A∧¬A”, then an arbitrary “B” follows from this. The paradox disappears right away if we understand that “A∧¬A" is the definition of an infinite universal. In fact, if we designate “A” as an object, then “¬A” will mean everything that is not “A”. That is, everything else in the universe. It would be strange if the statement “A∧¬A” did not imply the entire infinite diversity of our universal!

The formula “a∧¬a⊨∞” can well be proved in the system of natural inference, for example, by proving by contradiction:

Given: a∧¬a⊨ ∞

Hypothesis: 1. + ¬ (a∧¬a⊨∞)

2. ¬ (a∧¬a⊃∞) - from step 1 according to the deduction rule;

3. (a∧¬a) ∧¬∞ - from step 2 by the rule of negation of implication;

4. a∧¬a - from step 3 by the rule of deleting a conjunction;

5. a - from step 4 by the rule of deleting the conjunction;

6. ¬a - from step 4 by the rule of deleting a conjunction;

7. a∧¬a - from actions 5 and 6 according to the rule of introducing conjunctions;

8. (a∧¬a) ⊃¬ (a∧¬a⊃∞) - from actions 7 and 1 according to the rule of implication (or the rule of deduction);

9. a∧¬a⊃∞ - from step 8 by the rule of proof by contradiction.

Discussion

To date, a satisfactory solution to the question of formalizing logical implication does not exist. C. Lewis, who proposed replacing the “paradoxical” material implication with the so-called “strict implication”, could not solve this problem. Having rid himself of the material implication paradoxes, C. Lewis created the paradoxes of strict implication. In addition, Lewis could not remain within the framework of classical logic and did not quite successfully solve the tasks in modal logic.

At one time, W. Ackerman's theory was popular, according to which the logical implication of A⊨B is present where “B” is part of “A” - “B in A”. In other words, Ackerman assumes a logical implication where there is an implication with unified variables in the antecedent and consequent. However, this is only a partial solution to the problem, because a logical implication is possible (“If Euclid was wrong, and then the parallel lines intersect”) in statements where there are no such common variables. It is also possible to implicate logically in the absence of a “B in A” (“If no person is a beetle, therefore, no beetle is a person”).

But, with all its fairness, a new problem arises here: the problem of identifying the logical con-tent between “A” and “B”, the solution of which W. Ackerman could not offer within the framework of classical logic (in fact, W. Ackerman proposed solving the whole spectrum of these problems in relevance logic).

The idea was put forward of logical implication only between equivalent (identical) statements. But even this idea (with its unconditional truth) solves the question of logical implication only partially.

At different times, different scientists made attempts to close the question of formalizing logical implication (Voishvillo E.K. (1995), Zaitsev D.V., Sidorenko E.A. (2001), Lobovikov V.O. (2015), Rautley R., Muier R. (1981), Smirnov V. A. (1970), Tarski A. O (2015), Shalak V. I. O (2007), Ackerman W. (1956), Lewis C.I., Langford S. H. (1932), Newton I. (1994) and others), but that had not been possible so far.

Conclusion

1. The "paradox of material implication" is an imaginary paradox, since it is associated with linguistic confusion. Implication is not an operation of logical implication, but a consequence of a certain condition, which can be both necessary and sufficient.

Another “paradox” of implication is known as “anything follows from a contradiction”. This paradox cannot be taken seriously, because, firstly, it contradicts the laws of formal logic (and, therefore, should not be applied), but taken as a philosophical problem, it has its own solution in the form of an expression of infinity or psychological images.

2. The formalization of logical implication is a completely solvable problem, despite the fact tha.t until today it seemed impossible. We have shown that logical implication is formalizable. Perhaps we did it a little cumbersome and not quite aesthetically. But we proved that it is fundamentally possible to solve this problem

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